Background to Rock Roughness Equation

WATERWAY MANAGEMENT PRACTICES

Photo 1 – Rock-lined fish ramp

Photo 2 – Added culvert bed roughness

Introduction

Formulas such as the Strickler Equation have been commonly used for some time to estimate a Manning's n roughness value for rock lined channels; however, such equations are only appropriate for deepwater conditions where the flow depth is significantly greater than the rock size.

This fact sheet describes the development of an equation for the Manning's roughness of rock-lined surfaces operating in shallow water conditions. The equation has been developed from real world stream gauging, and is considered suitable for use in both shallow and deepwater conditions typically found in low to medium gradient streams, but **not** for highly turbulent whitewater.

Manning's roughness for deepwater conditions

To develop an equation for the purpose of estimating Manning's roughness at various relative flow depths it was first necessary to determine the asymptote of the equation, i.e. the Manning's roughness for rock-lined surfaces operating under deepwater conditions.

While developing his formula for fluid motion, Albert Strickler developed an equation for the selection of his roughness coefficient, 'k'. The Strickler formula (Equation 1) for fluid motion is similar to the Manning equation, except a coefficient 'k' is used instead of Manning's (1/n).

$$V = k R^{2/3} S^{1/2} \qquad (1)$$

where:
- V = mean flow velocity [m/s]
- k = roughness coefficient (Strickler coefficient)
- R = hydraulic radius [m]
- S = slope [m/m]

The associated roughness equation developed by Strickler is presented below as Equation 2.

$$k = 21.1/(d^{1/6}) \qquad (2)$$

where: d = mean size of gravel or boulders (i.e. d_{50}) [m]

Thus the Manning's coefficient (n) for deepwater conditions becomes:

$$n = ((d_{50})^{1/6})/21.1 \qquad (3)$$

where: n = Manning's roughness coefficient

Similar equations have been developed by Meyer-Peter & Muller, and Limerinos. These equations are presented below in their metric format.

Meyer-Peter & Muller $$n = ((d_{90})^{1/6})/26.0 \quad (4)$$

where: d_{90} = rock size for which 90% of rocks are smaller [m]

Limerinos (1970) $$n = \frac{(0.1129)R^{1/6}}{1.16 + 2.0\log(\frac{R}{d_{84}})} \quad (5)$$

Analysis of the data set used in this research showed the Meyer-Peter & Muller equation to produce more reliable estimates of the deepwater Manning's roughness values than the Strickler equation. Limerinos's equation was not found to produce satisfactory Manning's n roughness values for typical stream riffle systems as shown in Figure 1.

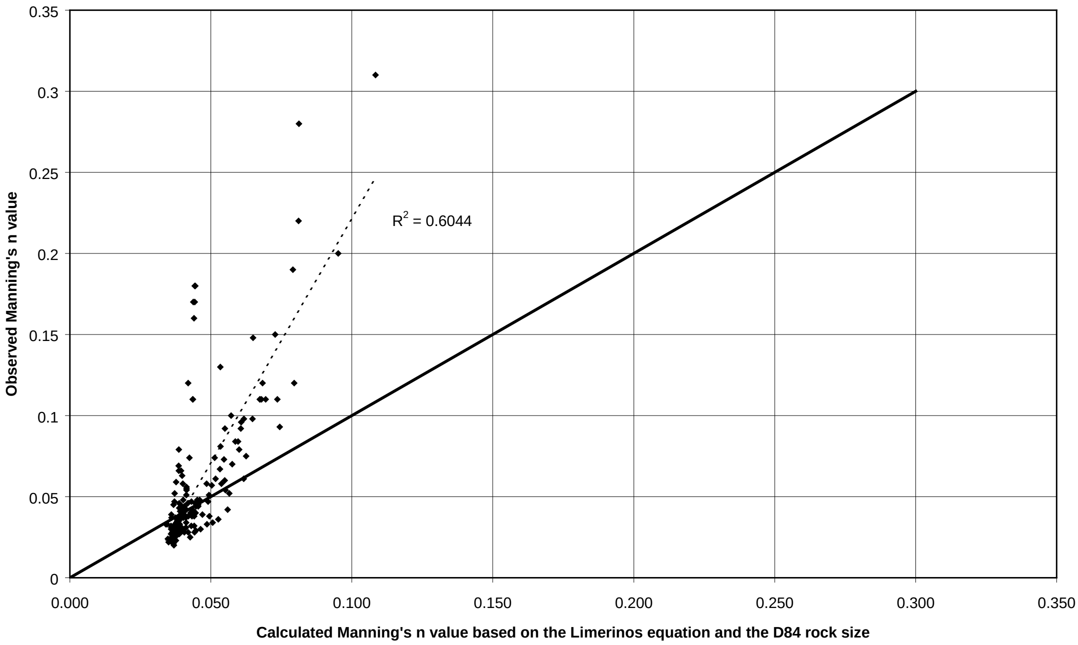

Figure 1 – Comparison between observed n-values and n-values determined from Limerinos (1970) based on d_{84} rock size

Data sources used in the assessment of shallow water roughness conditions

Data was obtained from the following sources for the assessment of shallow water flow conditions:

(i) An equation developed for wholly rough flow based on Nikuradse's experiments of uniform sand grain roughened pipes as reported in Vennard & Street (1976).

(ii) Flow resistance curves for Vegetal Retardance Group E assuming a uniform 25 mm grass blade length obtained from Department of Primary Industries (Queensland).

(iii) New Zealand stream gauging data presented in Hicks & Mason (1991) *Roughness Characteristics of New Zealand Rivers,* Water Resources Survey, DSIR Marine and Freshwater, Wellington, New Zealand. ISBN 0-477-02608-7.

Data from the first two sources was only used within the preliminary analysis. Only the New Zealand stream gauging data was used in the development of the final Manning's n roughness equation. The resistance curves for pipe flow and grassed channels were only used to provide a comparison with regard to determining the preferred form (i.e. shape) of the Manning's roughness equation.

Manning's roughness for uniform pipe roughness

Equation 6 represents the friction roughness equation for wholly rough flow (i.e. at high Reynolds numbers) in uniform sand grain roughened pipes (i.e. d_{50} approximates d_{90}).

Wholly rough flow:

$$\frac{1}{\sqrt{f}} = 1.14 + 2\log\left(\frac{D}{e}\right) \qquad (6)$$

where: f = pipe friction factor

D = pipe diameter [m]

e = effective sand grain roughness [m]

The friction factor (f) may be related to Manning's n using Equation 7.

$$\sqrt{f} = \frac{n\sqrt{8g}}{R^{1/6}} \qquad (7)$$

where: g = acceleration due to gravity [m/s^2]

Equation 8 was developed by combining Equations 6 and 7, and then rearranging.

$$n = \frac{R^{1/6}}{10.097 + 17.713\log\left(\frac{4R}{e}\right)} \qquad (8)$$

To provide a comparison with the roughness results from 25 mm grass, a roughness size 'e' of 25 mm was chosen. Table 1 presents the tabulated results for a range of flow depths.

Table 1 – Manning's roughness for uniform pipe roughness

e (mm)	R (mm)	R/e	n	n_o/n
25	6.25	0.25	0.043	0.479
25	12.5	0.5	0.031	0.652
25	25	1	0.026	0.780
25	50	2	0.023	0.873
25	75	3	0.022	0.916
25	100	4	0.022	0.937
25	150	6	0.021	0.964
25	200	8	0.021	0.977
25	300	12	0.021	0.990
25	400	16	0.020	0.996
25	600	24	0.020	1.000

The above analysis has only been provided to demonstrate a typical relationship between Manning's n and relative flow depth for wholly rough flow. The Manning's n for low values of 'R/e' are not expected to be correct.

Manning's roughness for a uniform grass cover

Roughness characteristics for a uniform grass surface were plotted for general information only. The retardance data was extracted from the Vegetal Retardance Curve for a Group E grass condition assuming 25 mm high grass on a 5% slope. The plotted data is presented in Table 2.

Table 2 – Manning's roughness for a uniform grass surface

e (mm)	R (mm)	R/e	n	n_o/n
25	50	2	0.058	0.431
25	60	2.4	0.047	0.532
25	80	3.2	0.038	0.658
25	100	4	0.032	0.781
25	150	6	0.028	0.893
25	200	8	0.025	1.0

Development of final data set from New Zealand stream gauging data

Hicks and Mason (1991) provides flow-gauging data for 78 locations throughout New Zealand. This stream gauging data was originally published to provide Manning's n data for rock lined channels operating under low flow conditions.

Of the 78 gauging stations, only data from those sites that met the following criteria were used in this investigation:

(a) d_{50} and d_{90} rock size data must be available for each site;

(b) the gauging station must lie in a relatively straight reach of the stream;

(c) the gauging station should not be located downstream of a sharp bend;

(d) primary channel roughness should result from bed roughness and should not be significantly influenced by bank or bed vegetation (in some channels, only low flow gauging records were used);

(e) the channel should have a uniform, near-rectangular cross section.

Only data points representative of shallow flow conditions (i.e. $R/d_{90} < 12$) were used. Deepwater conditions were assumed to exist for flow depth of $R/d_{90} > 12$.

Initially, deepwater Manning's n values (n_o) were determined by averaging the values obtained from both the Strickler and Meyer-Peter & Muller equations.

Strickler: $$n_o = ((d_{50})^{1/6})/21.1 \quad (9)$$

Meyer-Peter & Muller: $$n_o = ((d_{90})^{1/6})/26.0 \quad (10)$$

It is noted that the two equations produce equal 'n_o' values for $d_{50}/d_{90} = 0.2857$; in other words when $d_{90} = 3.5(d_{50})$.

At a number of gauging stations the deepwater Manning's n value was determined from the actual gauging results obtained for deepwater flow conditions. This technique could not be used in all cases because either the deepwater data was not available, or bank vegetation and/or channel irregularities were judged to have interfered with the deepwater gauging results. A summary of the adjusted deepwater Manning's n values is provided in Table 3.

Table 3 – Summary of adjusted deepwater Manning's roughness values (n_o)

Gauging Station	Adjusted deepwater Manning's roughness (n_o)
P30	0.024
P82	0.020
P94	0.029
P130	0.028
P238	0.033
P304	0.029

Data values with 'n_o/n' > 1.2 were considered to be unreliable and were removed from the data set. Such data values are likely to have been influenced by bank roughness and/or channel form as flow depth increased.

A summary of the data sets is provided in Table 4.

Table 4 – Summary of gauging station information

Stn [1]	d_{50} (mm)	d_{84} (mm)	d_{90} (mm)	n_{50}	n_{90}	n_o	d_{50}/d_{90}	Comments
P30	68	104	116	0.030	0.027	0.024	0.586	Medium width, straight, uniform constructed channel. Roughness equations appear to overestimate n_o.
P82	76	104	115	0.031	0.027	0.020	0.661	Wide to medium width channel with near uniform cross-section on a slight bend. Roughness equations appear to overestimate n_o.
P94	112	178	220	0.033	0.030	0.029	0.509	Wide, straight, near uniform cross-section. Strickler's equation based on d_{50} appears to overestimate n_o.
P102	52	116	150	0.029	0.028	0.028	0.347	Medium width channel with slightly irregular cross section.
P106	65	128	158	0.030	0.028	0.029	0.411	Straight, wide, uniform, constructed channel.
P110	46	80	92	0.028	0.026	0.027	0.500	Straight, uniform, constructed channel with uniform rock size and thick grass on banks.
P118	28	120	150	0.026	0.028	0.027	0.187	Medium channel width on a slight bend with irregular cross-section.
P122	24.3	78	90	0.026	0.026	0.026	0.270	Narrow channel with irregular cross-section located on a slight S-bend with trees on bank.
P130	104	200	234	0.033	0.030	0.028	0.444	Wide, straight channel with near uniform cross-section.
P146	56.3	182	255	0.029	0.031	0.030	0.221	Wide, straight channel with significant expansion.
P170	90	170	212	0.032	0.030	0.031	0.425	Wide, straight channel with uniform cross-section.
P186	23.5	104	126	0.025	0.027	0.026	0.187	Medium width, near uniform cross-section on a slight bend.
P194	27	150	195	0.026	0.029	0.028	0.138	Wide, irregular channel located on a slight bend with a slight expansion.
P198	33	90	111	0.027	0.027	0.027	0.297	Medium channel width with significant expansion.
P210	32	106	160	0.027	0.028	0.028	0.200	Medium width channel with irregular cross-section on a slight bend.
P238	46.3	91	115	0.028	0.027	0.033	0.403	Wide channel with near uniform cross-section.
P242	89	208	244	0.032	0.030	0.031	0.365	Wide, straight channel with near uniform cross-section.
P250	45	119	190	0.028	0.029	0.029	0.237	Straight, medium width channel with near uniform cross-section.
P258	70	200	280	0.030	0.031	0.031	0.250	Wide, near uniform channel on a slight bend.
P270	47	175	229	0.028	0.030	0.029	0.205	Wide channel with an irregular cross-section on a slight bend.
P278	78	212	300	0.031	0.031	0.031	0.260	Narrow, near uniform channel. The d_{90} rock size had to be estimated.
P290	69.3	168	218	0.030	0.030	0.030	0.318	Medium width channel on a slight meander with slight channel expansion and some bank roughness effects.
P304	16	124	200	0.024	0.029	0.029	0.080	Medium width channel of near-uniform cross-section. Effects of bank vegetation questioned. The d_{90} rock size had to be estimated.
P318	397	800	1080	0.041	0.039	0.040	0.368	Medium width channel of near uniform cross-section. Channel contains some very large rocks.
P324	94	258	350	0.032	0.032	0.032	0.269	Small to medium channel on a bend with slight channel irregularities. The d_{90} rock size had to be estimated.

Notes:

[1] Gauging stations named after the page number of data set within Hicks and Mason (1991).

Analysis of stream gauging data set

Following a detailed analysis of various best-fit equations the final formula was based on a logarithmic equation with an asymptote of:

'n' approaches 'n_o' as R/d_{90} approaches infinity

'n' approaches infinity as R/d_{90} approaches zero

However, it became obvious that a direct relationship could not be obtained between the relative roughness 'n_o/n' and relative flow depth 'R/d_{90}' as shown in Figure 2.

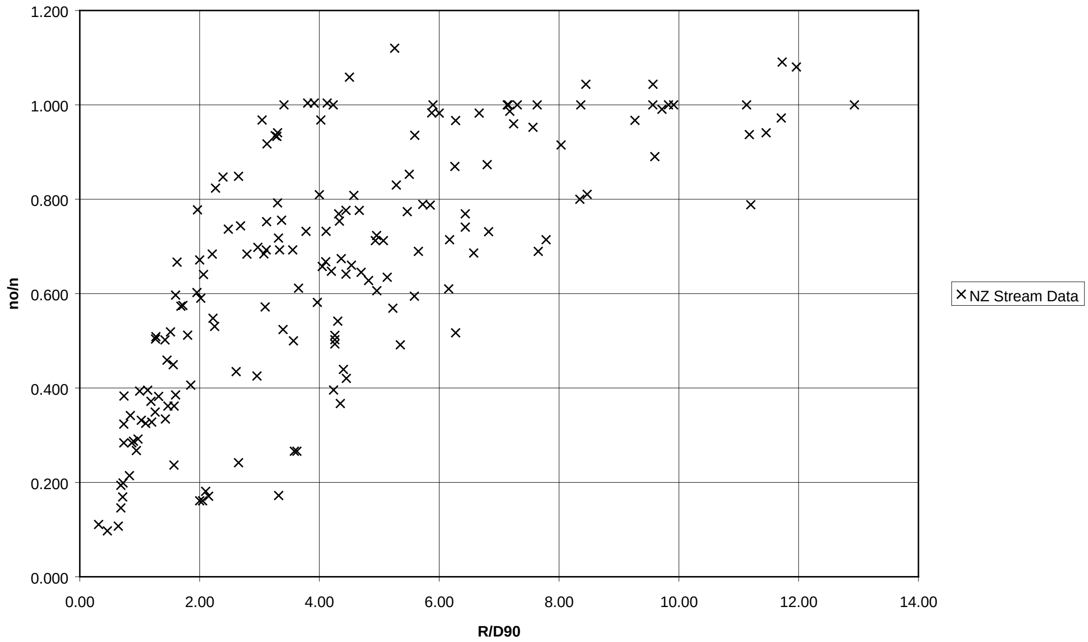

Figure 2 – Relative Manning's roughness for New Zealand stream data

Further analysis indicated that a better relationship could be obtained if the following asymptote was adopted:

'n_o/n' = 1.0 as $(R/d_{90})(d_{50}/d_{90})$ approaches infinity

'n_o/n' remains +ve as $(R/d_{90})(d_{50}/d_{90})$ approaches zero

Suitably rearranging the data set provided a best-fit equation in the form of Equation 11.

$$\left(\frac{n_o}{n}\right) = 1 - 0.3593^{(x)^{0.7008}} \qquad (11)$$

Adopting the Meyer-Peter & Muller equation for the determination of 'n_o' Equation 12 can be developed.

$$n = \frac{(d_{90})^{1/6}}{26(1 - 0.3593^{(x)^{0.7}})} \qquad (12)$$

where:
- x = $(R/d_{90})(d_{50}/d_{90})$
- R = hydraulic radius of flow over rocks [m]
- d_{50} = mean rock size for which 50% of rocks are smaller [m]
- d_{90} = rock size for which 90% of rocks are smaller [m]

Equation 12 can be tested by plotting the observed and calculated values of 'n_o/n' (Figure 3) and the observed and calculated values of Manning's n (Figure 4).

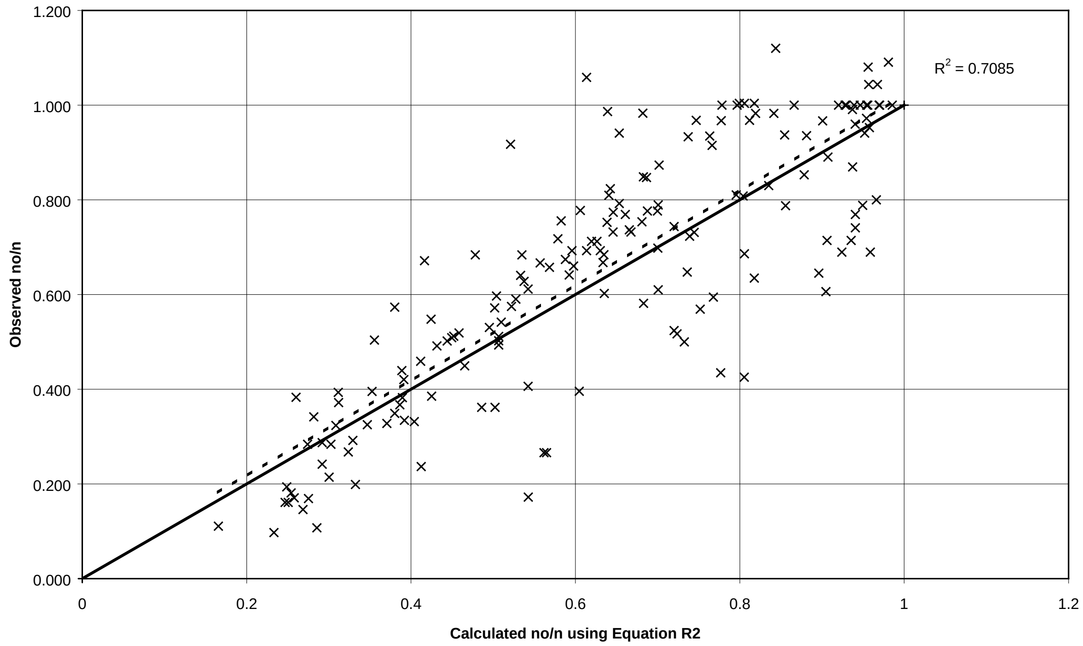

Figure 3 – Testing the suitability of Equation 12

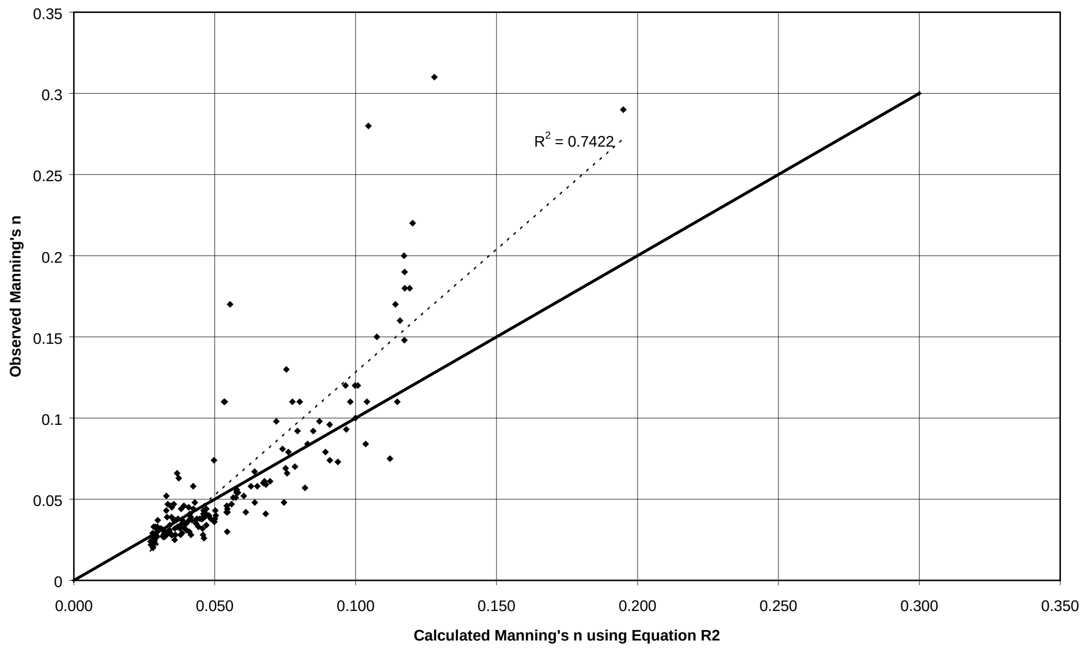

Figure 4 – Testing the suitability of Equation 12

Figures 3, 4 & 5 refer to 'Equation R2', which was the identity given to Equation 12 during the data analysis.

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PO Box 314
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Finally Equation 12 can be tested by plotting its output against the original plot of $(R/d_{90})(d_{50}/d_{90})$ versus 'n_o/n' as presented in Figure 5.

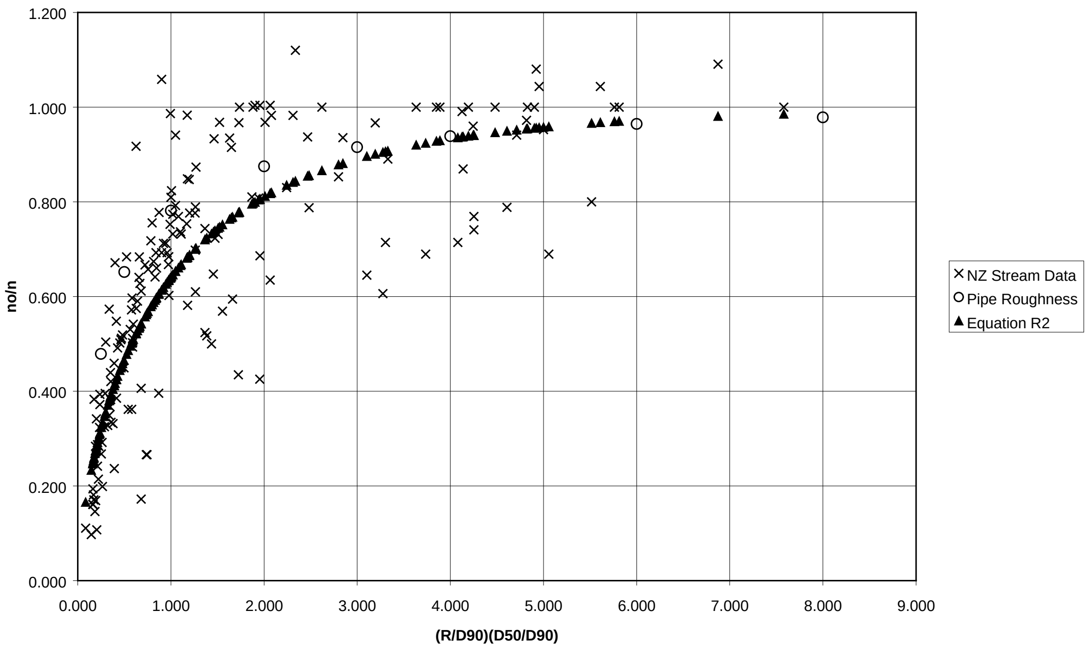

Figure 5 – Testing Equation 12 on the original stream gauging data set

As can be seen in Figure 5, Equation 12 produces more acceptable results at low and high values of $(R/d_{90})(d_{50}/d_{90})$, but produces slightly high values of Manning's n in the mid range area.

Comparing Figure 5 with Figure 4 also shows a significant reduction in the data spread at shallow water depths (i.e. low values of $(R/d_{90})(d_{50}/d_{90})$) indicating the importance of both the relative flow depth (R/d_{90}) and the rock size distribution (d_{50}/d_{90}) on hydraulic roughness.

Conclusions

Equation 12 is considered to produce significantly better estimates of the Manning's roughness of rock-lined surfaces in shallow water conditions compared to the use of traditional deepwater equations such as the Strickler, Meyer-Peter & Muller or Limerinos equations.

Given the high variability of Manning's n and the wide range of variables that are believed to influence the hydraulic roughness of a rock-lined channel, the results of Equation 12 are considered well within the limits of accuracy expected for Manning's n selection.

The data analysis showed that the Meyer-Peter & Muller equation (Eqn 4) produced more reliable estimates of the deepwater Manning's roughness values than the Strickler equation (Eqn 3). Possibly the choice between the two equations would come down to how reliable the determination of the d_{50} and d_{90} values were. If the estimate of d_{90} is not reliable, then it would be more appropriate to rely on the Strickler equation for the determination of the deepwater Manning's n value, and visa versa.

The data set used in the development of Equation 12 covered the range of values shown in Table 5. This table also contains the data range for the selected variables for which the calculated Manning's n value using Equation 12 fall within +/-10% of the observed Manning's n.

Table 5 – Data range used in determination of Equation 12

	d_{50} (mm)	d_{90} (mm)	R/d_{50}	R/d_{90}	n_o/n	d_{50}/d_{90}
Min (+/-10%)	16	90	2.31	0.73	0.284	0.080
Max (+/-10%)	112	350	55.6	12.0	1.080	0.661
Min (All data)	16	90	1.17	0.31	0.097	0.080
Max (All data)	397	1080	66.9	12.9	1.120	0.661